

Maths Vertical Curriculum

Maths: Introduction

Developed by early cultures, mathematics is one of the oldest forms of human knowledge. It is a cultural tool that can be used to measure, quantify, calculate, compute and express complex relationships and it is the basis of many sciences and most technology. Though terminology and some procedures vary culturally, the underlying principles in mathematics are universal. It is a way of thinking about the world that is innate in its elemental forms (e.g. unity, duality, parts of wholes and simple numbers) but as a cultural technique is only learned by most people through instruction. The link between the abstract symbols that are learned and the processes they represent require focused mental activity and reasoning, and because of its abstract nature, maths has to be carefully taught so that all children can grasp mathematical concepts and master mathematical processes.

Steiner Waldorf education always proceeds carefully, from the whole to the parts and from the tangible to the abstract so that all learners can follow the transitions. Mathematics is first experienced in concrete, practical ways through real-world activities. Then the relationships between quantities and their abstract numerical values and relationships (e.g. number bonds, time tables) are explored. Though maths is a way of thinking in itself, in the Waldorf curriculum it is always applied and practiced in relation to real questions of calculation in a wide range of fields from technology and crafts, dealing with money and economics as well as engineering and architecture. Maths is also a process that schools the imagination and flexibility of thinking through creative problem solving, but simultaneously exercises the will in seeing processes through their logical and necessary sequences.

Maths: Purpose of Study

Mathematics is a way of understanding the world. It is a creative and discipline with many applications, that has been developed over many centuries through exchange across many different cultures. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

Maths: Aims

The Waldorf curriculum for mathematics aims to ensure that all pupils:

- are skilled in the fundamentals of mathematics, applying their knowledge purposefully in a range of developmentally appropriate tasks
- develop conceptual understanding progressing from the concrete to the abstract, the simple to the complex, the whole to the parts and back again
- use their mathematical skills to solve increasingly complex problems over time through frequent and varied practice, and using a range of appropriate strategies

- reason mathematically by following a line of enquiry, discovering relationships and framing generalisations, developing an argument, justifying and proving using mathematical language

Long Term Curriculum Intents

Language and communication

- Using a wide mathematical vocabulary effectively, accurately and precisely
- Fluency in reading, writing and understanding mathematical symbols and formulae
- Representing and interpreting data in different forms and formats
- Representing, explaining and illustrating mathematical thinking and processes

Health and well-being

- Enjoyment of problem creation and solution

Senses

- Seeing the same thing in different ways, seeing from different perspectives

Imagination and play

- Having internal and mental models of numbers, shape, space and measure that support mathematical thinking
- Translating between abstract mathematical mental models and real life experiences or examples

Empathy

- Understanding the thinking, methods and routes to solutions of other people

Aesthetics

- Finding and appreciating elegant solutions to mathematical problems
- Appreciation of beauty in mathematical forms in the abstract, in nature and in man-made forms

Inquiry

- Explore patterns and not-patterns
- Explore mathematical ideas and experiment with different ways of finding solutions
- Being able to break down problems into small steps, and extract key information (decomposition and abstraction)
- Explore and experiment with shape, space and construction

Democratic participation and society

- Understanding the power of data and statistics in shaping and influencing political views
- Understanding how data and statistics can be used to manipulate people's political thinking

Lifelong learning

- Application of knowledge and skills in personal and professional life, e.g. mortgages, economics, personal/business accounts etc
- An understanding of mathematics and binary code as the basis of computer coding and programming
- Understanding the mathematical processes that calculators and computers are applied to

Future thinking

- Use mathematical thinking and interpretation of data to better understand risks to people and the world, and to imagine mitigations and solutions to these issues

Holistic thinking / Spirituality

- Appreciating the role of maths in spiritual experience, e.g. sacred geometry
- Using maths as a powerful language which can create connections with others across time and space

Judgement

- Forming judgements based on own understanding of data and statistics
- Evaluating routes to a solution, premise or theory

Maths Age-related Learning Opportunities for C1

Relevant Learning Descriptors

<p>Children should have the opportunity:</p> <p>Number</p> <ul style="list-style-type: none"> • To explore patterns and non-patterns, both man-made and in nature, including odd and even numbers. • To explore the quality of numbers and quantities to 20, how they can be represented with the fingers, and including how they are written in different numerical systems, how they can be ordered, partitioned, represented, combined and manipulated using the four operations. • To count and order numbers and quantities in a range of ways, in a range of situations and for a range of purposes, at least to 100 • To explore the importance of 10 in the Arabic number system, and the principles of place value, exchange and unitisation with numbers to 20 • To group and share objects and quantities in both concrete and abstract ways, including exploring the concept of 'half' • To experiment with equal groups of objects and quantities, exploring multiplication and division • To practise solving simple, single-step word or picture problems using manipulatives and pictures 	<p>Beginning Number</p> <p>Children can identify, reproduce and invent simple concrete patterns. They can differentiate between odd and even numbers. Children read, write and have an understanding of numbers to 20, counting accurately with correspondence and understanding cardinality. They count forwards and backwards to 100, order numbers and skip count in increments with regular patterns, knowing the most regular times-tables (e.g. 2s, 5s, 10s). Children can compare the magnitudes of different numbers, subitising smaller numbers, and having some fluency in composition, addition and subtraction of numbers to 20. They can exchange and unitise numbers to 20. Children group and share numbers, and partition shapes into simple fractions. Children solve word problems involving a single step using all four operations.</p> <p>Beginning Shape Space and Measure</p> <p>Children can name simple geometric shapes, draw them freehand and find examples of them in the environment. They can reproduce shapes and forms, continuing and extending patterns. Children are able to demonstrate their understanding of simple positional and directional language and symmetry in their work. They accurately compare lengths, heights, weights and capacities in practical activities. Children can recognise some coins, and count pence using a single, small denomination (e.g. 5 x 5p coins = 25p). Children indicate past, present or future using</p>
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<ul style="list-style-type: none"> • To discuss their thinking and methods with the teacher and their peers. <p>Shape, Space and Measure</p> <ul style="list-style-type: none"> • To explore straight lines and curves through movement and drawing • To explore using straight lines to draw regular-sided shapes and stars • To explore using convex and concave curves to draw waves, circles, ellipses, spirals, etc • To explore continuous patterns and sequences of shape • To explore relationships between shape and number • To experience simple positional and directional language, including horizontal/vertical/diagonal, and half/quarter turn • To explore symmetrical reflection • To compare lengths, heights, weights, capacities in practical ways • To encounter coins and notes as representations of quantity • To explore the everyday language of time to indicate past, present and future 	<p>simple, everyday language (e.g. yesterday, next week) and use the appropriate tense.</p>
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Maths Age-related Learning Opportunities for C2

Relevant Learning Descriptors

<p>Children should have the opportunity:</p> <p>Number</p> <ul style="list-style-type: none"> • To explore, describe and explain patterns and non-patterns, both concrete and numerical 	<p>Early Number</p> <p>Children can identify, describe, extend and invent patterns using concrete resources and numbers and can demonstrate their understanding of the difference between odd and even numbers. They count beyond 1000 in 1s, 10s, 100s and 1000s, reading and writing Arabic numerals to 100 and identifying the value of a numeral depending on its place, including</p>
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- To explore the number system beyond 1000, counting in 1s, 10s, 100s and 1000s
- To practise reading and writing numbers to 100 in Arabic form, including identifying the value of a numeral depending on place
- To practice addition and subtraction bonds to 20
- To explore and practice the four operations of number with numbers up to 100
- To explore the impact of place value on partitioning, adding and subtracting numbers
- To practise times tables in a range of ways
- To group and share objects and quantities in both concrete and abstract ways, exploring the familiar language of halves, quarters and thirds, and explaining and recording their thinking,
- To practise translating simple, single-step problems from words and pictures into mathematical equations (number sentences) and vice versa, exploring abstraction
- To discuss their thinking and methods with the teacher and their peers.

Shape, Space and Measure

- To practise mirroring forms made of straight lines and/or curves in both the horizontal and vertical axes, including the figure of eight, or lemniscate
- To practise transforming straight line forms into curved and vice-versa
- To extend continuous patterns and sequences of shape
- To explore the visualisation of multiplication as pattern
- To use simple positional and directional language, including horizontal/vertical/diagonal, and half/quarter turn

understanding this value in terms of tens and ones. Children have fluency with number bonds to 20 and can recite several of the more regular times-tables (e.g. 2s, 3s, 5s, 10s). They can calculate missing digits in simple equations with numbers to 100, working across 10s boundaries and using all four operations. Children can group and share objects and quantities, representing their thinking with concrete resources, pictures and numerals. They can extract the essential mathematical information from simple word and picture problems, solving the problem, explaining the calculation in their own words and recording their thinking numerically. They can do simple calculations with money, including calculating change.

Early Shape, Space and Measure

Children can draw reflections of forms made of straight lines and/or curves in both the horizontal and vertical axes, and can transform forms made of straight lines into those made of curves, and vice versa. Given the beginning of a pattern or sequence of shapes, they can extend this accurately. Children can describe aspects of their work using positional and directional language. During practical and craft activities, they measure lengths, heights, weights and capacities in useful and relatively accurate ways, using informal and/or non-standard measures. Children can recognise all coins and notes, can exchange equivalent amounts in different denominations, can count out a total amount using a mixture of denominations. They can compare durations of time using appropriate everyday language, e.g. minutes vs hours vs days, and can recognise the hours, half past and quarter past.

- To measure lengths, heights, weights, capacities in informal and/or non-standard ways during practical activities, e.g. cups, strides, knitting stitches
- To handle real coins and notes
- To explore the everyday language of the measurement of time – minutes, hours, days, months, years; and to tell the time to the hour, half hour and quarter.

Maths Age-related Learning Opportunities for C3

Children should have the opportunity:

Number

- To predict, continue and create sequential and numerical patterns
- To explore the number system beyond 10 000, counting forwards and backwards in 1s, 10s, 100s and 1000s from any number, and writing and ordering any numbers.
- To explore the use of known number facts on more complex mental calculations, e.g. near doubles, crossing 10s and 100s boundaries, place value strategies etc
- To explore the impact of place value on partitioning, adding and subtracting numbers
- To practise times tables in a range of ways as both multiplication and division, including exploration of square numbers
- To explore and practise simple written methods for the four operations of number with numbers up to 100
- To explore multiplication represented as arrays and in other pictorial ways, preceding calculation of area

Relevant Learning Descriptors

Developing Number

Children can predict, continue and create sequential and numerical patterns. They count forwards and backwards from any number in 1s, 10s, 100s and 1000s to beyond 10 000, reading, writing and ordering any numbers and describing the value of any digit in terms of 1s, 10s, 100s and 1000s. Children can perform mental addition and subtraction to 3 digits, using known number facts and their understanding of place value to support their working out. They can recite times-tables and answer multiplication and division questions to 12 x 12, recognising and recalling square numbers to 144. Children can use simple written methods to solve equations for all 4 operations with numbers to 100, explaining their reasoning verbally. They can represent multiplication in arrays. Children can use the language of halves, thirds and quarters accurately in different contexts, including o'clock, quarter past and half past on an analogue clock, grouping and sharing objects and quantities. They can solve word and picture problems by extracting the essential mathematical information, recording this in the form of an equation, and then solving it, explaining the calculation in their own words.

- To explore the language of halves, quarters and thirds in different contexts, e.g. shapes, numbers, time
- To practise translating single-step problems from words and pictures into mathematical equations and vice versa, practicing abstracting key information
- To discuss their thinking and methods with the teacher and their peers.

Shape, Space and Measure

- To practise copying and extending complex running forms, including forms which overlap, coil and intertwine
- To practise creating forms which include regular shapes, e.g. triangle, square, pentagon, hexagon
- To practise mirroring forms with a combination of horizontal, vertical and diagonal symmetries
- To use simple positional, directional and geometric language
- To explore the history of measurement, and the evolution of standard units
- To explore and practice estimation, measurement and comparison of length, weight and capacity
- To practice using the tools of measurement, e.g. rulers, balances, scales etc
- To relate mathematical problem solving skills to practical questions of measurement
- To explore how coins and notes can be combined to create amounts, be exchanged, and make change

Data Handling

Developing Shape, Space and Measure

Children can extend complex forms with overlapping, coiling and/or intertwined elements, identifying and naming common geometric shapes. They can draw symmetrical reflections of forms on the horizontal, vertical and/or diagonal axes, and use simple positional, directional and geometrical language to describe their work. Children can use appropriate tools to measure lengths, heights, weights and capacities of differing scales, using a range of non-standard and standard measures, understanding the history and conventions of the metric measuring system in the UK. They can use their mathematical knowledge and understanding to solve problems of measurement with all four operations. Children can use coins and notes to make totals and give change. They can tell the time to 5 minutes on an analogue clock.

Developing Data Handling

Children can add data in numbers, words and pictures to tables and charts.

- Explore the recording of data in tables and charts

Maths Age-related Learning Opportunities for C4

Relevant Learning Descriptors

Children should have the opportunity:

Number

- To explore number patterns which use one of the four operations
- To explore the number system beyond 100 000, counting forwards and backwards in 1s, 10s, 100s and 1000s from any number, and writing and ordering any numbers.
- To explore the addition and subtraction bonds of and within 50 and 100, e.g. $75 + 25$; doubling 100s and 50s
- To develop fluency in multiplication and division facts to 12×12 , including finding factors
- To explore and practice formal written methods for the four operations of number
- To explore fractions as parts of a whole, and discover how they can be represented, ordered, combined and manipulated
- To practice translating simple multi-step problems from words and pictures into mathematical equations and vice versa, practicing abstracting key information and exploring decomposition (breaking the problem down into parts)
- To discuss their thinking and methods with the teacher and their peers.

Shape, Space and Measure

Progressing Number

Children can identify and extend number patterns which use one of the four operations. They count forwards and backwards from any number in 1s, 10s, 100s and 1000s to beyond 100 000, reading, writing and ordering any numbers and describing the value of any digit in terms of 1s, 10s, 100s and 1000s. Children know number bonds of 100, and can fluently count in and add and subtract 100s and 50s. They know their multiplication tables and corresponding division facts to 12×12 both in and out of sequence, can find the factors of a given number, and identify common multipliers and factors for numbers. Children can use formal written methods to carry out calculations for all four operations with larger numbers. Children can read, write, order and manipulate simple fractions, adding and subtracting where the denominator is the same. They can identify equivalent fractions, simplify fractions and convert simple improper fractions to mixed numbers. Children can solve word problems involving multiple steps, identifying the essential mathematical information, and the steps needed, and explaining the calculation in their own words.

Progressing Shape, Space and Measure

Children can draw complex forms involving intertwining and plaiting. They draw regular shapes freehand, and can divide them into a given number of equal parts with relative accuracy. Children can use simple tools for measurement of length, height, weight and capacity with relative accuracy,

<ul style="list-style-type: none"> • To practice drawing intertwining and plaited motifs, e.g. knotwork, extending patterns and mirroring shapes • To explore geometrical drawing of regular shapes, and dividing shapes into equal parts • To explore and practice recording length, height, weight, capacity, time and money accurately • To explore and practice telling the time using minutes past and minutes to on an analogue clock • To practice applying mathematical knowledge and skill with the four operations in different contexts, e.g. money, measurement and time <p>Data Handling</p> <ul style="list-style-type: none"> • To explore recording data using tables and bar and pie charts 	<p>and record their measurements appropriately. They read and write the time using minutes past and minutes to and an analogue clock, and can record amounts of money using appropriate symbols, including a decimal point. Children can solve mathematical problems using units of measurement with all four operations.</p> <p>Progressing Data Handling</p> <p>Children can record data with relative accuracy in a table, bar chart and pie chart.</p>
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Maths Age-related Learning Opportunities for C5

Relevant Learning Descriptors

<p>Children should have the opportunity:</p> <p>Number</p> <ul style="list-style-type: none"> • To discover the rules governing numerical patterns • To explore the number system beyond 0.001, counting forwards and backwards in tenths, hundredths, thousandths from any number, and writing and ordering any numbers. • To explore the addition and subtraction bonds of and within 1 	<p>Competent Number</p> <p>Children can find and express the rule governing the creation of a pattern. They can generate a pattern using a rule of their choosing. Children can count forwards and backwards from any number in tenths, hundredths and thousandths, reading, writing and ordering any numbers and describing the value of any digit. They know the number bonds of 1 within tenths and hundredths. Children know their multiplication tables and corresponding division facts to 12 x 12 both in and out of sequence, can solve these equations quickly and confidently, including missing number problems, can</p>
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- To develop speed and fluency in multiplication and division facts to 12 x 12, including finding factors
- To develop fluency in formal written methods for the four operations of number
- To learn the rules that govern rounding
- To explore fractions of whole numbers
- To explore the connections between fractions, decimals and place value
- To explore using the four operations of number with fractions and decimals
- To practice translating multi-step problems from words and pictures into mathematical equations and vice versa, abstracting key information and using decomposition to break down complex problems into more manageable parts
- To discuss their thinking and methods with the teacher and their peers.

Shape, Space and Measure

- To be introduced to tools such as set square, ruler and compass to construct geometric figures
- To explore the calculation of perimeter and area
- To explore simple nets for 3D shape
- To explore Pythagoras' theorem, including the construction of different triangles
- To explore working with metric measurement of length, weight, volume and capacity, including using decimals
- To practice reading and writing the time in analogue and digital formats, including the 24 hour clock

find all the factors of a given number, and identify common multipliers and factors for numbers. They can round numbers to the nearest ten, hundred, tenth and hundredth. Children confidently, fluently and accurately use formal written methods to carry out calculations for all four operations with large numbers, and across the decimal point. They can find a fraction or a decimal of a whole number, translating between decimals and fractions. Children can perform all four operations with fractions, mixed numbers and improper fractions. They can apply all of their arithmetic skills to multi-step word problems, abstracting the key information and breaking the problem down into logical, solvable steps, and explaining the calculation in their own words.

Competent Shape, Space and Measure

Children can use a pair of compasses to draw accurate circles and divide them into a given number of parts. They can use perpendicular and angle bisection to construct a range of geometrical shapes, and also draw these freehand. Children can describe geometric shapes using accurate vocabulary, including for different types of triangle and angle. They can calculate the perimeter and area of a given shape, and draw freehand nets of 3D shapes. Children can demonstrate their understanding of Pythagoras' theorem in practical ways, using manipulatives and/or pictures. Children can first estimate, then weigh and measure accurately to decimal places, converting between units (e.g. cm to m; g to kg). They can read and write the time in both analogue and digital formats, and using both 12 and 24 hour conventions. Children can solve multi-step word problems in measurement of time, length, weight, capacity and volume using all four operations.

Competent Data Handling

Children can use simple coordinates to plot points on a grid and extract data from a grid. They can find information from a table or chart and use

<ul style="list-style-type: none"> To practice applying mathematical knowledge and skill with the four operations, including solving word problems, to problems involving time, length, weight, capacity and volume <p>Data Handling</p> <ul style="list-style-type: none"> To explore plotting points on a grid To practice extracting data from a table or chart 	<p>this information to solve a given problem, for example solving a word problem using a timetable.</p>
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Maths Age-related Learning Opportunities for C6

Relevant Learning Descriptors

<p>Children should have the opportunity:</p> <p>Number</p> <ul style="list-style-type: none"> To consolidate knowledge and understanding of the number system, including the use of indices/powers greater than 2; and ordering any whole numbers, common fractions and decimals to four places To explore the relationship between decimals, fractions and percentages To explore ratios To develop fluency, flexibility, and a range of strategies in mental and written calculations with all four operations, applied to integers, decimals, proper and improper fractions and mixed numbers To practice translating multi-step problems from words and pictures into mathematical equations and vice versa, abstracting key information and using decomposition to break down complex 	<p>Secure Number</p> <p>Pupils have a sound understanding of the number system, including indices/powers and place value. They can order any whole numbers and decimals to four places as well as common fractions, and compare decimals, fractions and percentages, using the appropriate mathematical symbols to denote relationships ($=$ \neq $<$ $>$ \leq \geq). Pupils can also express the relationship of two numbers with ratio notation, reducing to the simplest form. Pupils can use a range of strategies to perform both mental and written calculations with integers, decimals, proper and improper fractions and mixed numbers, using all four operations. This is supported by a fluent knowledge of number facts and times tables. Pupils can apply all of their arithmetic skills to multi-step word problems, including those involving time, length, weight, capacity and volume, abstracting the key information and breaking the problem down into logical, solvable steps, and explaining the calculation in their own words. Pupils can use their arithmetic knowledge and understanding in relation to simple business maths,</p>
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problems into more manageable parts, and including problems involving time, length, weight, capacity and volume

- To encounter simple business maths in practical contexts, e.g. simple and compound interest, VAT, and profit and loss.
- To discuss their thinking and methods with the teacher and their peers.

Shape, Space and Measure

- To explore the accurate construction of angles using compasses
- To explore the construction of leaf forms from triangles and circles.
- To explore the construction of similar, complementary, supplementary and other angles, and the associated terminology
- To explore the construction of triangles
- To explore the geometrical proof of sums of angles of triangles
- To explore congruent triangles and the four principal cases for congruency
- To explore other congruent shapes
- To explore coordinate axes
- To explore translations and movement properties of triangles and quadrilaterals
- To explore finding the perimeter and area of more complex shapes, and the volume of cuboids

Data Handling

- To explore the accurate collection and presentation of data using block graphs, line graphs and pictograms
- To use tables, graphs and diagrams to identify patterns and trends in data sets

calculating simple and compound interest, applying formulae such as VAT, price rises/decreases and discounts, and calculating profit and loss.

Secure Shape, Space and Measure

Pupils can construct shapes and forms with clean lines, using precise measurements and accurate angles. They can describe angles using appropriate terminology (obtuse, acute, reflex, right-angle, perpendicular, parallel, similar, complementary, supplementary etc). Pupils can use their understanding of formulae such as the sum of the angles of a triangle or the principal cases for the congruency of triangles to solve problems such as finding a missing angle and prove their answer. They can plot points and shapes onto coordinate axes, accurately rotating, translating and reflecting triangles and quadrilaterals. Pupils can find the perimeter and area of more complex shapes, and the volume of cuboids.

Secure Data Handling

Pupils can ask questions to generate data, and record answers in tables or charts. They can represent and display data using block graphs, line graphs or pictograms. Pupils can read, interpret and draw line graphs, extracting information to solve problems or draw conclusions. They can find the three types of average and the range of a set of integers. Pupils can identify and talk about relationships and patterns within sets of data, using tables, graphs and/or diagrams.

- To explore statistics

Maths Age-related Learning Opportunities for C7

Relevant Learning Descriptors

Children should have the opportunity:

Number

- To consolidate knowledge and understanding of the number system: ordering whole numbers, common fractions and decimals; converting between decimals, fractions and percentages; expressing relationships through ratios; identifying prime numbers and squares.
- To explore the powers and roots of numbers
- To consolidate fluency in mental and written calculations with all four operations, applied to integers, fractions, decimals, proper and improper fractions and mixed numbers
- To encounter recurring decimals and their conventions.
- To encounter negative numbers, introduced through business maths (see below), and then to explore the rules of calculating with negative numbers
- To explore the vocabulary used to describe elements of the number system (e.g. negative, natural and positive rational numbers, integers etc)
- To practice translating multi-step problems from words and pictures into mathematical equations and vice versa, abstracting key information and using decomposition to break down complex problems into more manageable parts, and including problems involving time, length, weight, capacity and volume.

Proficient Number

Pupils have a sound understanding of the number system, including place value. They can order any whole numbers, decimals and fractions, convert between decimals, fractions and percentages, express the relationship between two numbers as a ratio, identify prime numbers, calculate the square and estimate the square root of a number. Pupils can use a range of strategies to perform both mental and written calculations with integers, decimals, proper and improper fractions and mixed numbers, using all four operations and applying the rules for the correct order of operations. They can round numbers, including recurring decimals, to a given number of decimal places or significant figures. Pupils can calculate with negative numbers, and can describe numbers using mathematical vocabulary. They can apply all of their arithmetic skills to multi-step word problems, including those involving time, length, weight, capacity and volume, abstracting the key information and breaking the problem down into logical, solvable steps, and explaining the calculation in their own words. They can apply their knowledge and understanding of negative numbers to a simple balance sheet.

Proficient Algebra

Pupils can use and interpret algebraic notation, simplifying and manipulating algebraic expressions to maintain equivalence, and solving simple and linear equations.

- To explore negative numbers through profit/loss, debits and deficits
- To discuss their thinking and methods with the teacher and their peers.

Algebra

- To explore simple and linear equations, including the rules and conventions of using brackets, fractions and negative numbers, the correct order of operations, and associative/distributive properties
- To explore the making and transforming of formulae
- To explore simple set theory (e.g. through types of quadrilateral and their symmetries)

Shape, Space and Measure

- To explore the area of a circle and the value of π
- To explore the accurate calculation of the area of geometrical shapes, and any shape bounded by straight lines and circular arcs
- To explore translations and movement properties of given figures
- To explore Pythagoras' theorem
- To explore tangents to circles
- To explore perspective drawing

Data Handling

- To further explore statistics
- To explore probability through simple fractions and probability lines
- To explore the accurate collection and presentation of data using block graphs, line graphs and pictograms

Proficient Shape, Space and Measure

Pupils can calculate the perimeter and area of any shape. They can plot points and shapes onto coordinate axes, accurately rotating, translating and reflecting given figures. Pupils can use Pythagoras' theorem to solve problems involving right-angled triangles, and use known results to obtain simple proofs. They understand that a tangent forms a right angle with the radius at the point of intersection. Pupils can use the principles of perspective drawing to realistically depict three-dimensional objects in a two-dimensional plane.

Proficient Data Handling

Pupils can design useful questions and effective collection methods to generate, gather and record both qualitative and quantitative data. They can represent and display collected information in a range of appropriate ways, demonstrating their understanding of the conventions of tables, graphs and diagrams, e.g. axis, row and column labels, legends etc. Pupils can read, interpret and draw line graphs, extracting information to solve problems or draw conclusions. They can find the mean, median and mode of a set of numbers. Pupils can identify and talk about relationships and patterns within sets of data, using tables, graphs and/or diagrams.

- To use tables, graphs and diagrams to identify patterns and trends in data sets
- To explore negative numbers through profit/loss, debits and deficits

Maths Age-related Learning Opportunities for C8

Relevant Learning Descriptors

Children should have the opportunity:

Number

- To revise knowledge: ordering numbers, fractions and decimals; converting between decimals, fractions and percentages; showing relationships through ratios; identifying prime and square numbers; calculations with all operations, applied to all forms of number; round any number to a given value
- To explore the Base 2 number system: binary numbers, binary addition and the connections to computing
- To revise the vocabulary used to describe elements of the number system (e.g. negative, natural and positive rational numbers, integers etc)
- To practice translating multi-step problems from words and pictures into mathematical and algebraic equations and vice versa, abstracting information and using decomposition to break down problems into manageable parts, and including problems involving time, length, weight, capacity and volume.
- To explore balance sheets and mortgages

Mature, Independent Number

Pupils have a sound understanding of the number system, including place value. They can order any whole numbers, decimals and fractions, convert between decimals, fractions and percentages, express the relationship between two numbers as a ratio, identify prime numbers, calculate the square and estimate the square root of a number. Pupils can use a range of strategies to perform both mental and written calculations with integers, decimals, negative numbers, proper and improper fractions and mixed numbers, using all four operations. They can round any number to a given number of decimal places or significant figures. Pupils can use mathematical vocabulary appropriately. They can apply all of their arithmetic skills to multi-step word problems, including those involving time, length, weight, capacity and volume, abstracting the key information and breaking the problem down into logical, solvable steps, and explaining the calculation in their own words. Pupils can complete a simple balance sheet and solve problems involving the calculation of interest on mortgages.

Mature, Independent Algebra

Pupils can use and interpret algebraic notation, simplifying and manipulating algebraic expressions to maintain equivalence, and solving

- To discuss their thinking and methods with the teacher and their peers.

Algebra

- To further explore commutative, associative and distributive law and the expansion of brackets
- To explore the factors of the difference between the squares
- To explore graphs of straight lines and curves, and using these to solve simultaneous equations

Shape, Space and Measure

- To explore the accurate calculation of the volume and surface area of a range of geometrical shapes, and the relationship between weight and density
- To explore conic sections
- To explore the locus of line and plane,
- To explore the enlargement of given shapes
- To explore advanced properties of circles, e.g. orthogonal curves
- To further investigate Pythagoras' Theorem
- To explore internal and external angles of polygons
- To further explore spatial perspective drawing, including the golden section

Data Handling

- To further explore statistics, including the mean, median, mode, and range
- To explore probability through investigation of combination and permutation; recording, describing and analysing the frequency of

linear equations. Pupils can draw the graph of a linear equation. They can calculate the gradient and work out the y-intercept of a line from a graph and use these to build the equation of the line.

Mature, Independent Shape, Space and Measure

Pupils can calculate the volume and surface area of a range of shapes, and compare their densities. They can construct different types of conic section and plot the loci of lines and planes. Pupils can use coordinate axes to enlarge given figures. They can identify and apply circle definitions and properties, including centre, radius, diameter, circumference, tangent, arc, sector and segment, and can construct orthogonal curves. Pupils can use Pythagoras' theorem to solve problems, including finding the altitude of a given triangle. They can find the internal and external angles of polygons. Pupils can use the principles of perspective drawing to realistically depict three-dimensional objects in a two-dimensional plane.

Mature, Independent Data Handling

Pupils can find the mean, median, mode and range of a set of numbers. They can record, describe and analyse the frequency of outcomes of simple probability experiments, enumerating these using tables, grids and/or Venn diagrams. Pupils can identify and describe simple mathematical relationships between two variables in observational and experimental contexts, illustrating this using, for example, a scatter graph.

outcomes of an experiment, and enumerating possibilities systematically

- To use tables, graphs and diagrams to identify simple mathematical relationships between two variables